Non-Thermal Dark Matter, High Energy Cosmic Rays and Late-Decaying Particles From Inflationary Quantum Fluctuations

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Abstract

It has been suggested that the origin of cosmic rays above the GZK limit might be explained by the decay of particles, X, with mass of the order of 10^{12} GeV. Generation of heavy particles from inflationary quantum fluctuations is a prime candidate for the origin of the decaying X particles. It has also been suggested that the problem of non-singular galactic halos might be explained if dark matter originates non-thermally from the decay of particles, Y, such that there is a free-streaming length of the order of 0.1Mpc. Here we explore the possibility that quantum fluctuations might account for the Y particles as well as the X particles. For the case of non-thermal WIMP dark matter with unsuppressed weak interactions we find that there is a general problem with deuterium photo-dissociation, disfavouring WIMP dark matter candidates. For the case of more general dark matter particles, which may have little or no interaction with conventional matter, we discuss the conditions under which X and Y scalars or fermions can account for non-thermal dark matter and cosmic rays. For the case where X and Y scalars are simultaneously produced, we show that galactic halos are likely to have a dynamically significant component of X

scalar cold dark matter in addition to the dominant non-thermal dark matter component.

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1 Introduction

There have been a number of surprising observations made recently in cosmology. One has been the discrepency between simulations of galaxy halo formation and small-scale structure based on cold dark matter (CDM) and observations, which indicate that there is more small-scale structure than observed and that the halos are much less singular than predicted [1]. In particular, numerical simulations of CDM halo formation show that the abundance of dark matter subhalos and so dwarf galaxies within a galaxy should be the same as the abundance of subhalos within galactic clusters, whereas observations indicate that the abundance within galaxies is much less [2]. Various explainations have been put forward, including self-interacting dark matter [3], halos made of condensates of ultra-light scalar particles [4] and warm dark matter [5]. Recently it has also been suggested that dark matter could originate non-thermally from heavy particle (or topological defect) decay [6].

Another puzzle is the existence of ultra-high energy cosmic rays (UHECR) beyond the 10^{11} GeV GZK cut-off [7, 8]. A simple possible explaination of the UHECR observations would be to have a heavy decaying particle of mass $\approx 10^{12}$ GeV and lifetime longer than the age of the Universe, such that high energy proton, nuclei and photon primaries can originate within the 50Mpc mean free path for energies greater than the GZK cut-off [9]. A natural origin for such heavy decaying particles is from quantum fluctuations generated during inflation [10].

In this paper we consider whether the decaying particles which may explain non-singular halos and those which account for UHECR could both originate from quantum fluctuations during inflation. The halo problem is then assumed to be solved by a density of non-thermal dark matter (NTDM) coming from the decay of heavy particles which we label Y, such that the non-singular halos are a result of a free-streaming length ≈ 0.1Mpc [6], whilst the UHECR are explained by decaying particles X with masses of the order of 10^{12} GeV [8].

The paper is organized as follows. In section 2 we discuss the general conditions for solving the galactic halo problem via NTDM from heavy particle decay. We also briefly review the conditions required to account for UHECR via heavy particle decay. In section 3 we consider the special case of weakly interacting massive particle (WIMP) dark matter with unsuppressed weak interactions, in particular the conditions required to evade energy loss of the WIMPs via scattering from the thermal background whilst preserving primordial nucleosynthesis. In Section 4 we discuss whether the galactic halo problem and UHECR can be explained by X and Y particles generated during inflation and the possible observable consequences of this. In Section 5 we present our conclusions. In an Appendix we discuss the scattering cross-section and energy loss per scattering for a Majorana fermion scattering from thermal background particles.

2 General Conditions for NTDM and UHECR from Decaying Particles

2.1 Non-Thermal Dark Matter

The problem of singular galactic halos and excessive structure on small-scales can be solved if the dark matter has a free-streaming length of the order of 0.1Mpc [6]. To achieve this in the case of heavy dark matter particles, the dark matter must have a non-thermal origin, such as a decaying massive particle[1]. Suppose the dark matter particles originate from decay of a massive particle at $t = t_d$. Then the co-moving free-streaming length is given by [13]

$$\lambda_{FS} = \int_{a_d}^{a_{EQ}} \frac{v}{a^2 H} da \approx \frac{a_{NR}}{a_{EQ}^2 H_{EQ}} \left(1 + ln\left(\frac{a_{EQ}}{a_{NR}}\right)\right) \ , \tag{1}$$

where a_{EQ} is the scale factor at matter-radiation equality, a_d is the scale factor at t_d, v is the dark matter particle velocity, and we have used the fact that $v \propto a^{-1}$ once the freely propagating dark matter particles are non-relativistic. If we assume that the initial dark matter particle energy is βm_Y, we obtain

$$\lambda_{FS} \approx \frac{r_c}{a_{EQ}^2 H_{EQ}} \left(1 + ln\left(\frac{a_{EQ}}{r_c}\right)\right) \ , \tag{2}$$

[1] An alternative is to have warm dark matter, a thermal distribution of particles of mass approximately 1 KeV [5].

where [6]

$$r_c = \frac{\beta m_Y a_d}{m_\chi} \tag{3}$$

and m_χ is the dark matter particle mass. With $a = 1$ at present, r_c is equal to the present velocity of the dark matter particles. With $a_{EQ} = 4.3 \times 10^{-5}(\Omega_m h^2)^{-1}$, $\lambda_{FS} = 0.1$Mpc is obtained for [6]

$$r_c \approx 2.4 \times 10^{-8} \left(\frac{\lambda_{FS}}{0.1Mpc}\right) . \tag{4}$$

This is the condition for dark matter particles from Y decay to account for non-singular galactic halos. Using $a_d = \left(\frac{g(T_\gamma)}{g(T_d)}\right)^{1/3} \frac{T_\gamma}{T_d}$, this then fixes the Y mass as a function of the decay temperature,

$$m_Y \approx 4 \times 10^7 \; T_d \left(\frac{r_c}{10^{-7}\beta}\right) \left(\frac{g(T_d)}{g(T_\gamma)}\right)^{1/3} \left(\frac{m_\chi}{100 \text{ GeV}}\right) \text{ GeV} , \tag{5}$$

where $g(T)$ is the number of degrees of freedom in thermal equilibrium and T_γ is the photon temperature at present.

In addition, the dark matter particle density, assumed to be dominated by χ, must satisfy $\Omega_\chi \approx 0.3$. (We assume throughout a flat Universe with a cosmological constant $\Omega_\Lambda \approx 0.7$.) Assuming that the number of dark matter particles from each Y decay is ϵ, we have

$$\Omega_\chi = \frac{\epsilon m_\chi n_Y(T_\gamma)}{\rho_c} , \tag{6}$$

where ρ_c is the critical energy density and $n_Y(T_\gamma) = g(T_\gamma)T_\gamma^3/g(T_d)T_d^3$ is the Y number density at present in the absence of Y decays. The requirement $\Omega_\chi \approx 0.3$ will impose a constraint on the reheating temperature after inflation, T_R.

2.2 UHECR

The observed UHECR are not correlated with conventional sources of cosmic rays and are consistent with an isotropic distribution [8]. This is consistent with the idea that they originate from decaying massive particles [9]. The particles must have a mass $m_X \approx 10^{12}$ GeV and lifetime $\tau_X \approx 10^{16}(\xi_X/3 \times 10^{-4})\; yr$, where $\xi_X \approx \Omega_X/\Omega_\chi$ is the fraction of halo CDM in the form of X particles. For masses 10^{11} GeV $\lesssim m_X \lesssim 10^{13}$ GeV there is no conflict with the diffuse γ-ray background or positron flux in cosmic rays [8]. Therefore, as far as X particles generated during inflation are concerned, the only restriction is that $\Omega_X < 0.3$.

3 Non-Thermal WIMP Dark Matter

The conditions in Section 2 are quite general and apply to dark matter particles regardless of their interaction with particles in the thermal background. For the more specific case of Y particles decaying to WIMP dark matter and to a significant number of charged particles or photons, we must impose two other conditions.

The first condition is that the WIMPs from Y decay do not lose too much energy by scattering off of thermal background particles, since it is assumed that the WIMPs evolve by freely expanding after Y decay.

The condition for the WIMPs not to scatter is that, for the case of relativistic WIMPs, the scattering rate for WIMPs from thermal background particles should satisfy $\Gamma_{sc} \equiv n(T)\sigma_{sc}\Delta E < EH$, where $n(T) = \frac{1.2g(T)T^3}{\pi^2}$ is the number density of particles in the thermal background [13], E is the energy of the WIMP and ΔE is the energy lost by the WIMP per scattering with a particle in the thermal background. In the following we will consider WIMPs with masses of the order of m_W and unsuppressed weak interactions, such that the scattering with thermal background electrons and neutrinos is via t-channel Z^o exchange. Experimentally, Dirac fermions and scalars are excluded as dark matter particle in this case [11], so we consider the case of Majorana fermion WIMPs. In the Appendix we give the scattering cross-section for the scattering of relativistic Majorana fermions from thermal background particles and discuss the energy transfer per scattering.

The scattering cross-section times energy lost per scattering integrated over centre-of-mass scattering angle for Majorana WIMPs is (Appendix)

$$\sigma_{sc}\Delta E \approx \frac{2\pi\alpha^2 k_s}{E_\tau}, \quad k_s = \left(log\left(\frac{4E_\chi E_\tau}{m_Z^2}\right) - \frac{1}{4}\right) \quad ; \;\; 4E_\chi E_\tau > m^2_{\chi,\,Z} \;, \tag{7}$$

and

$$\sigma_{sc}\Delta E \approx \frac{32\pi\alpha^2 k_s^{'} E_\chi^4 E_\tau^3}{m_\chi^4 m_Z^4}, \quad k_s^{'} = \frac{40}{3} \quad ; \quad 4E_\chi E_\tau < m_{\chi,\,Z}^2 \ . \tag{8}$$

Here $\alpha = g_\chi g_\tau/4\pi \approx 0.006$ (with g_χ and g_τ the couplings of χ and the thermal background particles to Z^o (Appendix)) and $E_\tau \approx 3T$ is the energy of the thermal background particles.

For the case where $4E_\chi E_\tau > m_{\chi,\,Z}^2$ ($T_d > \mathrm{Max}(m_Z^2,\ m_\chi^2)/(3\beta m_Y)$), no-scattering imposes the constraint

$$m_Y \gtrsim \frac{4\alpha^2 g(T_d) k_s M_{Pl}}{3\pi\beta k_{T_d}} \approx 9\times 10^{15}\left(\frac{1}{\beta}\right)\left(\frac{\alpha}{0.006}\right)^2\left(\frac{g(T_d)}{100}\right)^{1/2}\left(\frac{k_s}{10}\right)\ \mathrm{GeV}\ , \tag{9}$$

where $k_T = \left(\frac{4\pi^3 g(T)}{45}\right)^{1/2}$. This is ruled out if Y particles are to be generated by quantum fluctuations during inflation, since particles with $m_Y > H_I \approx 10^{13}$ GeV cannot be generated during inflation [10], where H_I is the expansion rate during inflation [2].

For $4E_\chi E_\tau < m_{\chi,\,Z}^2$ ($T_d < \mathrm{Min}(m_Z^2,\ m_\chi^2)/(3\beta m_Y)$), we obtain an upper bound on the decay temperature

$$T_d \lesssim \left(\frac{1}{864 k_s^{'} g(T)}\right)^{1/4} \frac{k_{T_d}^{1/4} m_Z m_\chi}{\alpha^{1/2}\beta^{3/4} m_Y^{3/4} M_{Pl}^{1/4}}\ . \tag{10}$$

Numerically this gives

$$T_d \lesssim \frac{5.2\times 10^{-3}}{\beta^{3/4}}\left(\frac{0.006}{\alpha}\right)^{1/2}\left(\frac{m_\chi}{100\ \mathrm{GeV}}\right)\left(\frac{m_Z}{90\ \mathrm{GeV}}\right)\left(\frac{100\ \mathrm{GeV}}{m_Y}\right)^{3/4}\left(\frac{10}{g(T_d)}\right)^{1/8}\ \mathrm{GeV}\ . \tag{11}$$

(We keep m_Z explicit in this in order to check the effect of increasing the mass scale of the gauge interaction.)

Combined with the free-streaming constraint Eq. (5), we find that in general no-scattering requires that

$$m_Y \lesssim 10.4\left(\frac{0.006}{\alpha}\right)^{2/7}\left(\frac{1}{\beta}\right)\left(\frac{r_c}{10^{-7}}\right)^{4/7}\left(\frac{m_\chi}{100\ \mathrm{GeV}}\right)^{8/7}\left(\frac{m_Z}{90\ \mathrm{GeV}}\right)^{4/7}\left(\frac{g(T_d)}{10}\right)^{5/42}\ \mathrm{TeV} \tag{12}$$

[2]It may be possible to generate such heavy particles via preheating after inflation [12]

and

$$T_d \lesssim 1.6\times10^{-4}\left(\frac{0.006}{\alpha}\right)^{2/7}\left(\frac{10^{-7}}{r_c}\right)^{3/7}\left(\frac{m_\chi}{100\ \mathrm{GeV}}\right)^{1/7}\left(\frac{m_Z}{90\ \mathrm{GeV}}\right)^{4/7}\left(\frac{10}{g(T_d)}\right)^{3/14}\ \mathrm{GeV}\ . \tag{13}$$

Thus in order to both evade scattering and act as a source for non-thermal WIMP dark matter, $m_Y \lesssim 10$TeV and $T_d \lesssim 2\times10^{-4}$GeV is necessary.

In the above we have imposed the condition that the WIMPs effectively lose no energy through scattering with thermal background particles. However, if $T_d < \mathrm{Min}(m_Z^2,\ m_\chi^2)/(3\beta m_Y)$ then the rate of loss of energy decreases as the particles lose energy, since $\Gamma_{sc}\Delta E/E \propto E_\chi^3$. So in this case, if T_d is larger than the upper bound in Eq. (13), it may appear that it is possible for the WIMPs to lose energy and stop scattering from the thermal background whilst still remaining non-thermal, so allowing T_d to evade the upper bound Eq. (13) from no-scattering. However, it is easy to see that this is not the case: the effect of WIMPs losing energy to the thermal background is to effectively reduce the initial energy βm_Y at T_d until the no-scattering condition is satisfied. However, the upper bound from no-scattering and free-streaming, Eq. (13), is β independent, so reducing β will not allow T_d to evade the upper bound.

A second condition, which applies if $T_d \lesssim T_{nuc} \approx 1$MeV and if there is a significant number of photons produced in the Y decay cascade, is that the decaying Y particles should not dissociate light elements produced during nucleosynthesis [14]. We will impose the constraint following from the non-dissociation of deuterium D, using the conservative bound based on assuming that a fraction f_γ of the energy from Y decays is entirely in the form of threshold 2.3MeV photons, which is the maximum number possible from the cascade decay [15]. D non-dissociation then requires that

$$n_{2.3}\, p(2.3\ \mathrm{MeV}) < n_D\ , \tag{14}$$

where $n_{2.3} \equiv f_\gamma \rho_Y(T_d)/2.3MeV$ is the number density of 2.3MeV photons, n_D is the number density of D nuclei and $p(E)$ is the probability of a photon of energy E photo-dissociating a D nucleus [15]

$$p(E) = \frac{n_D \sigma_D}{n_e \sigma_T}\ . \tag{15}$$

σ_D is the D photo-dissociation cross-section, σ_T is the total photon scattering cross-section from background particles (for threshold photons these are given by $\sigma_D = 3$mb and $\sigma_T = 125$mb [15]) and n_e is the electron density. Using $n_e = n_B = \eta_B s(T)$, where n_B is the baryon number density, η_B is the baryon number to entropy ratio and $s(T) \equiv \frac{2\pi^2 g(T) T^3}{45}$ is the entropy density, this results in a T_d-independent bound on the Y mass

$$m_Y < 2.3 \, \frac{2\pi^2 g(T_\gamma) T_\gamma^3}{45} \frac{\sigma_T}{\sigma_D} \frac{\epsilon m_\chi \eta_B}{f_\gamma \Omega_\chi \rho_c} \text{ MeV} . \quad (16)$$

Numerically this gives

$$m_Y \lesssim 0.26 \, \frac{\epsilon}{f_\gamma h^2} \left(\frac{\eta_B}{5 \times 10^{-11}} \right) \left(\frac{0.3}{\Omega_\chi} \right) \left(\frac{m_\chi}{100 \text{ GeV}} \right) \text{ GeV} , \quad (17)$$

where critical density is $\rho_c = 7.5 \times 10^{-47} h^2 \text{ GeV}^4$. However, since $\epsilon m_\chi \leq m_Y$, this results in the condition

$$f_\gamma \lesssim \frac{2.6 \times 10^{-3}}{h^2} \left(\frac{\eta_B}{5 \times 10^{-11}} \right) \left(\frac{0.3}{\Omega_\chi} \right) . \quad (18)$$

Therefore unless only a very small fraction of the Y mass ends up in electromagnetic final states, deuterium dissociation rules out Y decay after nucleosynthesis.

Taken in conjunction with the free-streaming and no-scattering constraint, which requires that $T_d \lesssim 2 \times 10^{-4}$ GeV, we see that in most cases the deuterium constraint rules out WIMPs with unsuppressed weak interactions as non-thermal dark matter. If we wish to retain WIMPs with unsuppressed weak interactions as dark matter we must have Y particles decaying dominantly to WIMPs with little or no electromagnetic states produced in the decay. This is difficult, since in general a neutral weakly interacting particle will come in an $SU(2)_L$ representation together with electromagnetically charged particles. So we would require a model in which the mass of the Majorana WIMP χ^o was lighter than its charged $SU(2)_L$ partners $\chi^\pm$, such that $2m_{\chi^o} < m_Y < 2m_{\chi^\pm}$. In this case $Y \rightarrow 2\chi^o$ would be kinematically allowed but $Y \rightarrow \chi^+\chi^-$ disallowed. Since the mass splitting of the $SU(2)_L$ representation must come from $SU(2)_L$ breaking and so must be at most of the order of m_W, the Y mass must also be of the order of m_W in any NTDM WIMP model with unsuppressed weak interactions.

Alternatively we can consider dark matter particles which interact more weakly with the thermal background than WIMPs with unsuppressed weak interactions and raise the T_d upper bound above T_{nuc}. This can be done either by reducing the strength of the WIMP coupling to the exchanged particle or by increasing the mass scale of the exchanged particle. From Eq. (13) we see that this requires that $(m_Z/\alpha^{1/2})$ is increased by a factor of about 25 to have an upper bound on T_d greater than 1MeV, where we consider m_Z to represent the mass scale of a general exchanged particle. From the free-streaming condition Eq. (5) we see that if $T_d \gtrsim$ 1MeV then the decaying Y particle must have a mass greater than 40TeV for m_χ of the order of 100 GeV, or more generally that $m_Y/m_\chi \gtrsim 400$.

In [17] it has been suggested that dark matter neutralinos could act as non-thermal dark matter. They find that for the case of neutralinos with unsuppressed Z^o couplings (Wino limit), NTDM is ruled out, in agreement with the results derived here, but that for neutralinos with suppressed Z^o couplings (Bino limit), which scatter from the thermal background via slepton exchange, and with heavy sleptons of mass of the order of 1TeV, it is marginally possible to have non-thermal neutralino dark matter with a free-streaming length of 0.1Mpc.

4 Particle Densities From Quantum Fluctuations

In the previous sections we have considered the conditions for the dark matter particles from Y decay to account for non-thermal dark matter. We now consider the origin of the heavy particles which may be responsible for NTDM and UHECR. In the following we will consider χ and Y particles without imposing restrictions coming from their interactions with conventional matter. For example, it is possible that the non-thermal dark matter particles and decaying Y particles could belong to a hidden sector which couples only very weakly if at all to conventional matter. In this case T_d could take any value without disrupting nucleosynthesis.

We will make the simplest assumptions regarding inflation, namely that the expansion rate H is fixed during inflation, with inflation followed by a coherently oscillating inflaton matter dominated period characterized by a reheating temperature T_R.

4.1 X and Y Scalars

We will consider the amplitude of the quantum fluctuations to be such that only the mass term in the scalar potential plays a role i.e. we neglect self-interactions. We also consider the case where $\phi = 0$ at the beginning of inflation (where ϕ represents the X or Y scalars), so that there is no significant contribution to the number density of scalars from an initial classical expectation value for ϕ. For $m < H_I$, the scalars may be considered massless during inflation.

During each interval $\delta t \approx H_I^{-1}$, the scalar field receives a quantum fluctuation on horizon scales $\delta\phi \approx H_I/2\pi$, which is thereafter stretched beyond the horizon as a classical fluctuation. Therefore after N e-foldings of inflation, the mean squared magnitude of the classical scalar field as seen on sub-horizon scales will be [10]

$$\delta\phi^2 \approx \frac{NH_I^2}{4\pi^2} \tag{19}$$

as a result of the random walk of the scalar field due to quantum fluctuations. Once inflation ends and the Universe enters the inflaton matter dominated regime, the super-horizon wavelength modes begin to re-enter the horizon. We denote the expansion rate when the fluctuation of wavelength λ re-enters the horizon as H_λ. The evolution of the modes then depends on whether they re-enter before or after $H = m$. The modes obey the equation of motion

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - \mathbf{k}^2\delta\phi = -m^2\delta\phi \ , \tag{20}$$

where $\mathbf{k}$ is the wavenumber of the mode. Modes entering at $H_\lambda \leq m$ have $\delta\phi$ constant until $H = m$, after which they oscillate about $\delta\phi = 0$ with $\delta\phi \propto 1/a^{3/2}$. Modes entering at $H_\lambda > m$ are additionally suppressed, since their equation of motion is initially dominated by the $\mathbf{k}^2$ term. As a result their energy density evolves as $\delta\phi \propto 1/a$ until $\mathbf{k}^2 \propto a^{-2} < m^2$, after which the mass term dominates and the modes evolve as $\delta\phi \propto 1/a^{3/2}$. The average number density of ϕ particles at $H < m$ will therefore be given by

$$n \approx m < \delta\phi^2 >= n_{(H_\lambda < m)} + n_{(H_\lambda > m)} = (N_T - N_m) m \delta\phi_c^2 + m < \delta\phi^2_{(H_\lambda > m)} > \ \ , \quad (21)$$

where $< ... >$ denotes spatial average, $\delta\phi^2_{(H_\lambda > m)}$ is the total contribution of modes with $H_\lambda > m$, N_m is the number of e-foldings before the end of inflation at which modes re-entering at $H = m$ leave the horizon, N_T is the total number of e-foldings of inflation ($N_T \gtrsim 65$) and $\delta\phi_c$ is the amplitude of each $H_\lambda < m$ mode. In general, a mode re-entering at H during inflaton matter domination exits the horizon during inflation at

$$N(H) = \frac{1}{3} ln \left(\frac{H_I}{H} \right) \ . \quad (22)$$

With $H_I \approx 10^{13}$ GeV and $H = m \gtrsim 100$ GeV, for example, $N_m \lesssim 8$. Thus to a reasonable approximation we can ignore the contribution of modes with $H_\lambda > m$ and simply consider $N_m = 0$ in Eq. (21). In this case the gradient terms in the $\delta\phi$ equation of motion can be neglected and all modes simply oscillate about the minimum of the potential such that $\delta\phi \propto a^{-3/2}$ once $H \leq m$, whlist remaining constant at $H > m$. Therefore during matter domination by the inflaton ($H \propto a^{-3/2}$), $\delta\phi_c$ is given by

$$\delta\phi_e \approx \frac{H}{m} \frac{H_I}{2\pi} \ . \quad (23)$$

Thus during inflaton matter domination, the number density of ϕ scalars is

$$n(H) \approx \frac{N_T}{4\pi^2} \frac{H^2 H_I^2}{m} \ . \quad (24)$$

The number density at temperatures $T \leq T_R$ is then

$$n(T) \approx \frac{N_T \pi g(T)}{45} \frac{T_R T^3 H_I^2}{m M_{Pl}^2} \ . \quad (25)$$

4.2 X and Y Fermions

Quantum production of fermions during inflation is possible only if the fermions are massive [16]. The resulting density during matter domination has been estimated to be [16]

$$n = C_\alpha m H^2 \ , \quad (26)$$

where $C_\alpha \approx 3 \times 10^{-3}$ for a transition from inflation to matter domination. In general, the density of scalars will be much larger than that of fermions of the same mass, by a factor

$$\frac{N_T}{4\pi^2 C_\alpha} \left(\frac{H_I}{m}\right)^2 \gg 1 \ . \tag{27}$$

As a result, if one has scalars and fermions with similar decay rates to dark matter particles (for example, supersymmetric partners), then the number of dark matter particles produced will be determined by the scalars.

4.3 NTDM from Scalars

In order to account for NTDM from the decay of Y scalars we require that $\Omega_\chi \equiv \epsilon n(T_\gamma) m_\chi/\rho_c = 0.3$. Using Eq. (25) we find that this requires that

$$T_R \approx \frac{45 m_Y M_{Pl}^2 \rho_c \Omega_\chi}{\epsilon m_\chi N_T \pi H_I^2 g(T_\gamma) T_\gamma^3} \ . \tag{28}$$

Numerically this gives

$$T_R \approx 280 \left(\frac{60}{N_T}\right) \left(\frac{h^2}{\epsilon}\right) \left(\frac{m_Y}{100 \text{ GeV}}\right) \left(\frac{100 \text{ GeV}}{m_\chi}\right) \left(\frac{10^{13} \text{ GeV}}{H_I}\right)^2 \left(\frac{\Omega_\chi}{0.3}\right) \text{ GeV} \ . \tag{29}$$

4.4 UHECR from Scalars

The requirement that $\Omega_X < 0.3$ implies the m_X-independent constraint

$$T_R \lesssim 0.3 \frac{45 M_{Pl}^2 \rho_c}{N_T \pi H_I^2 g(T_\gamma) T_\gamma^3} \ . \tag{30}$$

Numerically this gives

$$T_R \lesssim 280 h^2 \left(\frac{10^{13} \text{ GeV}}{H_I}\right)^2 \left(\frac{60}{N_T}\right) \text{ GeV} \ . \tag{31}$$

Thus a low reheating temperature is necessary in order to allow UHECR to be explained by scalars generated by quantum fluctuations.

4.5 Relationship Between X and χ Densities from Scalars

It is important to note that the mass density of X and Y scalars from quantum fluctuations is approximately the same, since the number density Eq. (25) is inversely proportional to m. (This is in contrast with the case of fermions, where the number density is proportional to m.) Therefore, in general, the present density of dark matter particles from Y scalar decay is related to the density of X scalars by

$$\frac{\Omega_\chi}{\Omega_X} = f_\Omega \frac{\epsilon m_\chi}{m_Y} \ , \tag{32}$$

where f_Ω, as discussed below, parameterizes the uncertainty in the number density of scalars from quantum fluctuations. Since $\epsilon m_\chi \leq m_Y$, in order to have $\Omega_\chi > \Omega_X$ and so have dark matter primarily in the form of non-thermal χ dark matter, we must have $f_\Omega > 1$. If $\Omega_\chi > \Omega_X$ then $m_Y/\epsilon \geq m_\chi > m_Y/f_\Omega\epsilon$, and so for ϵ and f_Ω not very different from 1 m_χ and m_Y must be of the same order of magnitude.

f_Ω parameterizes the fact that the number density Eq. (25) has an uncertainty. For example, if the total number of e-foldings of inflation N_T is much larger than 65, then most of the number density in Y scalars would be due to superhorizon modes, resulting in a coherently oscillating field $\delta\phi(t)$ in the Universe. The initial amplitude of this field is determined by a 1-dimensional random walk of the field during inflation due to quantum fluctuations. For n steps of size $\pm a$, the probability of a particle being at x is described by the normal distribution

$$P(x)dx = \frac{e^{-\frac{x^2}{2a^2 n}} dx}{(2a^2 n\pi)^{1/2}} \ . \tag{33}$$

The root mean square value is given by $x_{rms}^2 = na^2$. In our case $x \equiv \delta\phi$, $a \equiv H_I/(2\pi)$ and $n \equiv N_T$. As an estimate of the range of values f_Ω can reasonably take we integrate Eq. (33) and exclude the range of values $(0, |\delta\phi_1|)$ and $(|\delta\phi_2|, \infty)$ for which the probability is less than 0.316, chosen to give the range of values of f_Ω which have a probability of 0.8. This gives $|\delta\phi_1| = 0.41|\delta\phi_{rms}|$ and $|\delta\phi_2| = |\delta\phi_{rms}|$. (Note that the best estimate of the mean value of $|\delta\phi|$ is somewhat smaller than $|\delta\phi_{rms}|$. If we take the value for which the probability of being larger or smaller is 0.5, then the mean value is $0.7|\delta\phi_{rms}|$.) The number density $n \propto \delta\phi^2$ can therefore reasonably take values in the range 0.34 to 2.04 times the mean density. So in this case $f_\Omega = n_Y/n_X$ can reasonably be in the range 0.17 to 5.9, with the probability of being smaller or larger than these values being $0.316^2 = 0.1$. (This is independent of the definition of the mean density.) So long as $f_\Omega > 1$, the dark matter density can be primarily due to non-thermal χ dark matter. However, we still expect to find a significant contribution to the total dark matter density from X scalars. This is important, since in this case galactic halos will be composed of a combination of free streaming non-thermal dark matter particles and a smaller but possibly dynamically significant component of conventional cold dark matter due to X scalars. This may result in different predictions for halo formation and small-scale structure as compared with the limiting cases of pure non-thermal dark matter or pure cold dark matter.

4.6 NTDM from Fermions

In this case, in order to account for $\Omega_\chi = 0.3$ we require that

$$T_R = \frac{45 M_{Pl}^2 \rho_c \Omega_\chi}{4\pi^3 g(T_\gamma) C_\alpha T_\gamma^3 \epsilon m_\chi m_Y} \ . \tag{34}$$

Numerically this gives,

$$T_R = 4.3 \times 10^{24} \, \frac{h^2}{\epsilon C_\alpha} \left(\frac{100 \text{ GeV}}{m_\chi}\right) \left(\frac{100 \text{ GeV}}{m_Y}\right) \left(\frac{\Omega_\chi}{0.3}\right) \text{ GeV} \ . \tag{35}$$

Since $m_Y \lesssim H_I \approx 10^{13}$ GeV for fermions generated from quantum fluctuations, there is a lower bound on the reheating temperature as a function of m_Y,

$$T_R \gtrsim 1.4 \times 10^{16} \, \frac{h^2}{\epsilon} \left(\frac{3 \times 10^{-3}}{C_\alpha}\right) \left(\frac{100 \text{ GeV}}{m_\chi}\right) \left(\frac{\Omega_\chi}{0.3}\right) \text{ GeV} \ . \tag{36}$$

For example, we consider two mass scales for χ and Y particles which are of particular interest: the weak scale m_W and the mass scale 10^{12-13} GeV associated with X particles and H_I. Since the largest possible reheating temperature after inflation is $T_R \approx (H_I M_{Pl})^{1/2} \approx 10^{16}$ GeV, the case $m_\chi \sim m_W$ is only marginally compatible with Y fermions from quantum fluctuations and only if $m_Y \approx 10^{13}$ GeV. The free-streaming constraint, Eq. (5), then implies that the Y fermions decay at temperature

$$T_d \approx 2.5 \times 10^5 \left(\frac{10^{-7}\beta}{r_c}\right) \left(\frac{g(T_\gamma)}{g(T_d)}\right)^{1/3} \left(\frac{100 \text{ GeV}}{m_\chi}\right) \left(\frac{m_Y}{10^{13} \text{ GeV}}\right) \text{ GeV} \ . \tag{37}$$

(In this case WIMPs are ruled out as dark matter by the no-scattering constraint, Eq. (9).) For the case of very large χ mass, $m_\chi \approx 10^{13}$ GeV, the reheating temperature must satisfy

$$T_R \gtrsim 1.4 \times 10^5 \, \frac{h^2}{\epsilon} \left(\frac{3 \times 10^{-3}}{C_\alpha} \right) \left(\frac{10^{13} \text{ GeV}}{m_\chi} \right) \left(\frac{\Omega_\chi}{0.3} \right) \text{ GeV} \,, \tag{38}$$

where we have imposed $m_Y \approx 10^{13}$ GeV since $m_Y \geq \epsilon m_\chi$, whilst the Y fermions decay at

$$T_d \approx 2.5 \times 10^{-6} \left(\frac{10^{-7}\beta}{r_c} \right) \left(\frac{g(T_\gamma)}{g(T_d)} \right)^{1/3} \left(\frac{m_Y}{m_\chi} \right) \text{ GeV} \,. \tag{39}$$

(In this case WIMPs are ruled out as dark matter by the nucleosynthesis constraint.)

4.7 UHECR from Fermions

In this case the constraint $\Omega_X < 0.3$ implies that

$$T_R < 0.3 \, \frac{45 M_{Pl}^2 \rho_c}{8\pi^3 C_\alpha T_\gamma^3 m_X^2} \,. \tag{40}$$

Numerically this gives

$$T_R \lesssim 1.4 \times 10^7 h^2 \left(\frac{3 \times 10^{-3}}{C_\alpha} \right) \left(\frac{10^{12} \text{ GeV}}{m_X} \right)^2 \text{ GeV} \,. \tag{41}$$

Thus a wide range of reheating temperatures is consistent with UHECR from X fermions.

5 Conclusions

We have considered the conditions under which non-singular galactic halos and UHECR can be explained by decaying particles produced during inflation by quantum fluctuations. For the case of WIMP non-thermal dark matter from decaying particles, the requirement that WIMPs with unsuppressed weak interactions do not lose energy by scattering from the thermal background combined with the requirement that their free-streaming length is of the order of 0.1Mpc implies that the Y particles decay at temperatures less than that of nucleosynthesis. Thus unless Y particles can decay to

WIMPs without producing a significant number of photons in the cascade, WIMPs with unsuppressed weak interactions are ruled out by photodissociation of deuterium. It may be marginally possible to suppress the weak interactions of the WIMPs sufficiently that the Y particles can decay above 1 MeV, for example in the case of supersymmetry with Bino dark matter and with heavy sleptons of mass ~ 1TeV [17].

An alternative possibility is to consider the case where the dark matter particles and Y particles belong to a sector interacting only very weakly if at all with conventional matter, such that there is no danger to nucleosynthesis from Y decay products. In the case where UHECR and NTDM originate simultaneously from X and Y scalars generated by quantum fluctuations, the present mass density of X scalars will naturally be of the same order of magnitude as that of the non-thermal dark matter. In this case we expect the dark matter in the galactic halo to be a combination of free-streaming non-thermal dark matter and conventional X scalar cold dark matter, which could alter the predictions for galactic halo and small-scale structure formation from the case of pure non-thermal dark matter. This would allow an observational test of the idea that decaying scalar particles generated during inflation can explain both UHECR and non-singular galactic halos.

Appendix. Majorana Fermion Scattering Cross-Section

The scattering cross-section in the CM frame is

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{1}{64\pi^2 s}\frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}|\overline{\mathcal{M}}|^2 \ , \tag{42}$$

where $\mathbf{p}_{i,\,f}$ are the initial and final three-momenta and s is the CM energy squared. We consider a Majorana fermion χ of mass m_χ scattering from a massless thermal background fermion τ. For simplicity we will consider a head-on collision of χ with τ along the x-direction; this can generally be arranged by boosting the frame in the cases where the collision is at an angle, resulting in shifts in the energy and momenta by factors of the order of 1. The weak interactions are described by

$$\mathcal{L}_{int} = \frac{g_\chi}{2}\overline{\chi}\gamma^\mu\gamma_5\chi Z_\mu + g_\tau\overline{\tau}\gamma^\mu(1-\gamma_5)\tau Z_\mu \ . \tag{43}$$

In numerical estimates we use for g_χ and g_τ the value of the $\nu_L \nu_L Z^o$ coupling from the Standard Model, $g_\tau = g\chi/2 = g_2/4Cos\theta_W = 0.19$, where g_2 is the $SU(2)_L$ gauge coupling. The spin-averaged amplitude squared for t-channel Z^o exchange is then

$$|\overline{\mathcal{M}}|^2 = \frac{16 g^2 g^{'\,2}}{(k^2 - m_Z^2)^2}[(p_1.q_1)(p_2.q_2) + (p_1.q_2)(p_2.q_1) + m_\chi^2 (q_1.q_2)] \ , \tag{44}$$

where $p_{1,2}$ are the initial and final χ 4-momenta, $q_{1,2}$ are the initial and final τ 4-momenta and $k^2 = (q_2 - q_1)^2$. For $E_\chi \gg E_\tau, \ m_\chi$ we find

$$|\overline{\mathcal{M}}|^2 = \frac{16 g^2 g^{'\,2}}{(k^2 - m_Z^2)^2}\frac{8E_\tau^4 \gamma^4}{\eta^2}[1+2\eta(1+Cos\theta)+2\eta^2(1+Cos^2\theta)+\frac{m_\chi^2}{2E_\tau^2}\frac{\eta^2}{\gamma^2}(1-Cos\theta)] \ , \tag{45}$$

where θ is the scattering angle in the CM frame, η and γ are defined by

$$\eta^2 = \frac{E_\chi^2 E_\tau^2}{(m_\chi^2 + 2E_\chi E_\tau)^2}, \qquad \gamma^2 = \frac{E_\chi^2}{(m_\chi^2 + 4E_\chi E_\tau)} \ , \tag{46}$$

and

$$k^2 = \frac{-8E_\chi^2 E_\tau^2 (1 - Cos\theta)}{m_\chi^2 + 4E_\chi E_\tau} \ . \tag{47}$$

Also $s = m_\chi^2 + 4E_\chi E_\tau$. The energy loss per scattering in the LAB frame (the rest frame of the thermal background) for a Majorana fermion scattering at angle θ in the CM frame from a thermal background particle, in the limit $E_\chi \gg m_\chi, E_\tau$, is given by

$$\Delta E = \frac{2E_\chi^2 E_\tau (1 - Cos\ \theta)}{(m_\chi^2 + 4E_\chi E_\tau)} \ . \tag{48}$$

In the limit $4E_\chi E_\tau > m_{\chi,\,Z}^2$ the cross-section times energy lost per scattering integrated over scattering angle is then

$$\sigma_{sc}\Delta E \equiv \int d\Omega \left(\frac{d\sigma}{d\Omega}\right)_{CM} \Delta E(\theta) \ \approx \frac{g^2 g^{'\,2}}{8\pi}\frac{k_s}{E_\tau} \quad ; \quad k_s = \left(log\left(\frac{4E_\chi E_\tau}{m_Z^2}\right) - \frac{1}{4}\right) \ . \tag{49}$$

In the limit $4E_\chi E_\tau < m_{\chi,\,Z}^2$ the cross-section times energy lost per scattering is

$$\sigma_{sc}\Delta E \approx \frac{2g^2 g^{'\,2}}{\pi}\frac{k_s^{'} E_\chi^4 E_\tau^3}{m_\chi^4 m_Z^4}, \quad ; \quad k_s^{'} = 40/3 \ . \tag{50}$$

References

[1] A.A.Klypin, A.V.Kravtsov, O.Valenzuela and F.Prada, *Ap. J.* **522** (1999) 82; B.Moore, T.Quinn, F.Governato, J.Stadel and G.Lake, *Ap. J.* **499** (1998) L5; B.Moore, *Nature* **370** (1994) 629; A. Burkert, *Ap. J.* **447** (1995) L25; S.S.McGaugh, W.J.G. de Blok, *Ap. J.* **499** (1998) 41; J.F.Navarro and M.Steinmetz, *Ap. J.* **528** (2000) 607; W.J.G. de Blok and S.S.McGaugh, *Mon. Not. R. Astron. Soc.* **290** (1997) 533; P.Salucci and A.Burkert, *Ap. J.* **537** (2000) L9; P.Salucci and A.Borriello, *astro-ph/0011079.*

[2] B.Moore, *astro-ph/0103100.*

[3] D.N.Spergel and P.J.Steinhardt, *Phys. Rev. Lett.* **84** (2000) 3760.

[4] P.J.E.Peebles, *Phys. Rev.* **D62** (2000) 023502; astro-ph/0002495; J.Goodman, astro-ph/0003018; *see also* J.McDonald, *Phys. Rev.* **D48** (1993) 2575.

[5] C.J.Hogan and J.J.Dalcanton, *Phys. Rev.* **D62** (2000) 063511; J.J.Dalcanton and C.J.Hogan, astro-ph/0004381; J.Sommer-Larsen and A.Dolgov, astro-ph/9912166; V.K.Narayanan, D.N.Spergel, R.Dave and C-P.Ma, astro-ph/0005095; P.Colin, V.Avila-Reese and O.Valenzuela, *Ap. J.* **542** (2000) 622.

[6] W.B.Lin, D.H.Huang, X.Zhang and R.Brandenberger, astro-ph/0009003.

[7] K.Greisen, *Phys. Rev. Lett.* **16** (1966) 748; G.T.Zatsepin and V.A.Kuzmin, *Pis'ma Zh. Eksp. Teor. Fiz* **4** (1966) 114.

[8] D.J.Bird et al, *Phys. Rev. Lett.* **71** (1993) 3401; *Ap. J.* **424** (1994) 491; *ibid.* **441** (1995) 144; N.Hayashida et al, *Phys. Rev. Lett.* **73** (1994) 3491; S.Yoshida et al, *Astropart. Phys.* **3** (1995) 105.

[9] V.Berezinsky, M.Kachelriess and A.Vilenkin, *Phys. Rev. Lett.* **79** (1997) 4302; M.Birkel and S.Sarkar, *Astropart. Phys.* **9** (1997) 297; S.Sarkar, hep-ph/0005256.

[10] D.J.Chung, E.W.Kolb and A.Riotto, *Phys. Rev. Lett.* **81** (1998) 4048; *Phys. Rev.* **D59** (1999) 023501; V.Kuzmin and I.I.Tkachev, *Phys. Rev.* **D59** (1999) 123006; G.Felder, L.Kofman and A.Linde, hep-ph/9909508.

[11] G.Jungman, M.Kamionkowski and K.Greist, *Phys. Rep.* **267** (1996) 195.

[12] E.W.Kolb, A.Linde and A.Riotto, *Phys. Rev. Lett.* **77** (1996) 4290.

[13] E.W. Kolb and M.S. Turner, *The Early Universe* (Addison-Wesley, Reading MA, USA, 1990).

[14] S.Sarkar, *Rep. Prog. Phys.* **59** (1996) 1493.

[15] D.Lindley, *Mon. Not. R. Astron. Soc.* **188** (1979) 15P.

[16] V.A.Kuzmin and I.I.Tkachev, *JETP Lett.* **68** (1998) 271; *Phys. Rep.* **320** (1999) 199.

[17] J.Hisano, K.Kohri and M.Nojiri, *hep-ph/0011216*.